AXIAL FORCES OF A V-BELT CVT
PART I : THEORETICAL ANALYSIS

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A simple analytical formula is derived for the axial forces of a V-belt CVT drive. For the driver pulley axial force, belt tension is assumed to be constant by self-locking. For the driven pulley axial force, the whole arc of contact between the belt and the pulley is divided into two regions; (1) inactive and (2) active area. The sliding angle ϕ for the belt friction is approximated as $\phi=0$ in the active area. It is found that the results of the new formula are in close agreement with the exact solution. The new formula is suggested for the V-belt CVT design due to its simplicity and sufficient accuracy.

Key Words: V-Belt CVT, Axial Force, Speed Ratio, Torque Load

1. INTRODUCTION

V-belt CVT(Continuously Variable Transmission) is a transmission having a speed ratio that can be varied continuously over its allowable speed range. Its speed ratio may take on any value between its operational limit, i.e., an infinite number of ratios are possible. A gearbox transmission, on the other hand, has a discrete number of fixed speed ratios. This property of the V-belt CVT gives a getter fuel economy compared with that of classical gearbox transmission. Besides, the V-belt CVT has many advantages such as compact, light weight, low manufacturing cost because it has a relatively small number of parts.

Transmission mechanism of the V-belt CVT is shown on Fig.1. Each of driver and driven pulley consists of a fixed and a movable pulley. The fixed pulleys are fixed on the shafts and the movable pulleys are able to move in the axial direction on the shafts. Continuously variable transmission can be achieved by control of the pulley axial distance between the fixed and the movable pulleys. If the movable pulley of the driver shaft is moved towards the fixed pulley, the V-belt is forced to be pushed in the radial outward direction, which causes the belt pitch diameter to increase. Since the belt length and the center distance between the shafts are fixed, the blet pitch diameter of the driven pulley decreases. Therefore, the speed ratio decreases in a continuous manner. Any desired speed ratio can be obtained by control of the pulley axial displacement. Since the pulley axial displacement is controlled by axial force on the driver and the driven pulleys, an accurate relationship between the speed ratio and the axial force is required to maintain an optimum driving condition. Also, the axial forces are directly related with the belt tension. If the belt tension and associated axial forces are kept only as high as necessary to prevent slip at all load levels, then an enormous improvement in belt life will result compared to tension set for maximum design power. Therefore, we can say that it is an integral part of the V-belt CVT design to obtain an accurate relationship between the axial force and torque load for given speed ratios.

Worley(1955) introduced an experimental formula for the axial forces of the V-belt CVT. Since his equation is simple and convenient to use, it has been widely used in industry in spite of the discrepancies with experimental data. Oliver(1973) used the Worley's equation for the driver pulley axial force with a different coefficient of friction. For the driven pulley axial force, he derived an equation based on belt friction forces in the radial and tangential direction. But Oliver regarded a whole contact arc between the belt and the pulley as an active area. Obviously this is not true.

For the V-belt, radial motion causes a radial component of friction force. Introduction of the radial dimension and a noncircular belt trajectory complicates the problem enormously. In an extensive treatment of the problem, Gerbert(1972 and 1974) presented a rigorous set of seven nonlinear differential equations describing force and slip behavior of a V-blet. He considered a belt movement of the radial and tangential direction, noncircular curvature of the belt and

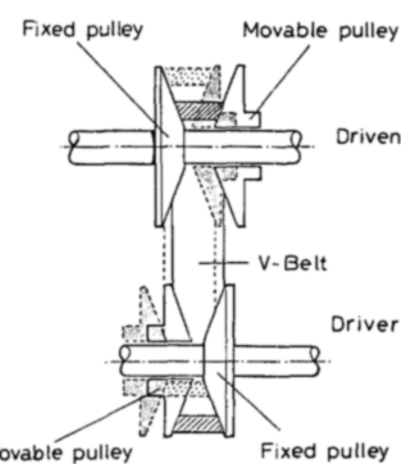

Fig. 1 Principle of a V-belt CVT

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belt elastic properties. His solutions are numerical integration of approximate equations and considered to be exact solutions. However, as noted by Gerbert, the existence of an "orthogonal point" makes the numerical work difficult to avoid a singular point. Besides, his equations require a tedious time consuming numerical work even by the today's point of view.

Therefore, in this paper, a simple anaytical formula is derived for the axial forces of a V-belt CVT drive which fits the exact solution and can be used as a design formula in industry.

2. THEORETICAL ANALYSIS

2.1 Assumptions to Develop Analytical Equations

In the analysis of a V-belt drive, the contact arc between the belt and the pulley should be divided into two areas ; (1) inactive area and (2) active area(Kim, et. al, 1987, Kim and Marshek, 1988). In the inactive area, there is only a static friction between the belt and the pulley and the belt tension does not change. In the active area, kinetic friction occurs because of a belt slip and this friction causes the belt tension to change. The kinetic friction occurs in the opposite direction of the belt movement, while the belt moves in the radial and tangential direction in the active area. For a flat belt drive, these inactive and active areas always exist in the driver and driven pulley when a torque load is transmitted. However, for a V-belt drive, the two distinctive regions, the inactive and active areas, are not found in the driver pulley.

Figure 2 and 3 show the experimental results by Lutz(1965)

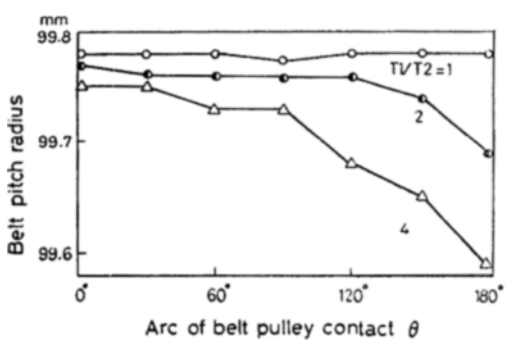

Fig. 2 Belt pitch radius versus contact arc for driven pulley (From Ref. Lutz, 1965)

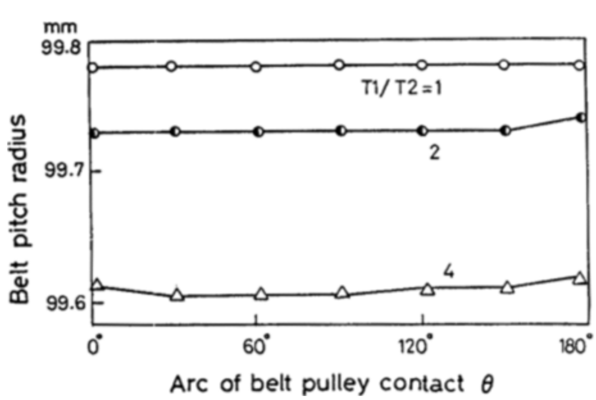

Fig. 3 Belt pitch radius versus contact arc for driver pulley (From Ref. Lutz, 1965)

for the belt pitch radius of the driven and driver pulley with various tension ratios. Tension ratio is defined as the ratio of a tight side belt tension T_1, to a slack side belt tension T_2 as T_1/T_2. For a tension ratio $T_1/T_2=1$, that is, no torque load case, the curve of the belt pitch radius remains constant. This means that there is no change in the belt tension and all the contact area is inactive. As the tension ratio increases, in other words, the torque load increases, the contact arc is divided into two areas. The belt pitch radius is almost constant in the inactive area and decreases in the active area for the driven pulley(Fig.2). The decrease of the belt pitch radius in the active area results from the increase of belt tension. As the belt tension increases, the belt penetrates into the V-shape wedge of the pulley and this causes the belt pitch radius to decrease. However, for the driver pulley, the curves of the belt pitch radius remain almost constant throughout the contact area and both the inactive and the active area can not be distinguished (Fig.3). This can be explained by "self-locking".

For the driver pulley, the belt pitch radius remains costant with constant belt tension T_1 in the inactive area. In the active area, theoretically, the blet tension should change from T_1 to T_2 and the corresponding belt pitch radius should increase because the belt tension decreases from T_1 to T_2. However, the increase of the belt pitch radius is constrained by static friction forces on both sides of the V-belt, i.e., self-locking. At self-loking state, the static friction on both sides of the belt is greater than the radial component of the belt tension. Thus, the belt can not move in the radial outward direction and there is only a static friction in the radial direction. Since there is no friction in the tangential direction, the belt tension remains unchanged as T_1.

Proceeding from these foundations, the following assumptions are made in order to derive simple analytical equations for the axial forces:

(1) For the driver pulley, belt tension remains constant as T_1 for the entire arc of contact by self-locking.
(2) For the driven pulley, the arc of contact is divided into two areas: (1)inactive area where the belt tension is constant and (2)active area where the belt tension changes due to the belt movement in the radial and tangential direction.
(3) The coefficient of friction μ between the belt and the pulley is constant.
(4) The influence of belt bending moment is negligible.

2.2 Driver Pulley Axial Force

Free body diagram of a driver pulley belt element is shown on Fig.4. From the assumption of self-locking for the driver pulley, there is only a radial friction on the belt element and no change in the belt tension. Radial static equilibrium for the belt element of length ds, as shown in Fig.4, requires that

$$F_c+2Nds\sin\frac{\alpha}{2}+2\mu Nds\cos\frac{\alpha}{2}-2(T_1+C)\sin\frac{d\theta}{2}=0 \quad (1)$$

where C is a centrifugal tension of a belt, i.e., $C=(w/g)v^2$ and F_c is a centrifugal force for belt element ds, $F_c=(w/g)v^2d\theta$.

Simplifying the Eq. (1) gives

$$T_1d\theta=2Nds(\sin\frac{\alpha}{2}+\mu\cos\frac{\alpha}{2}) \quad (2)$$

The driver pulley axial force for the belt element Fds is

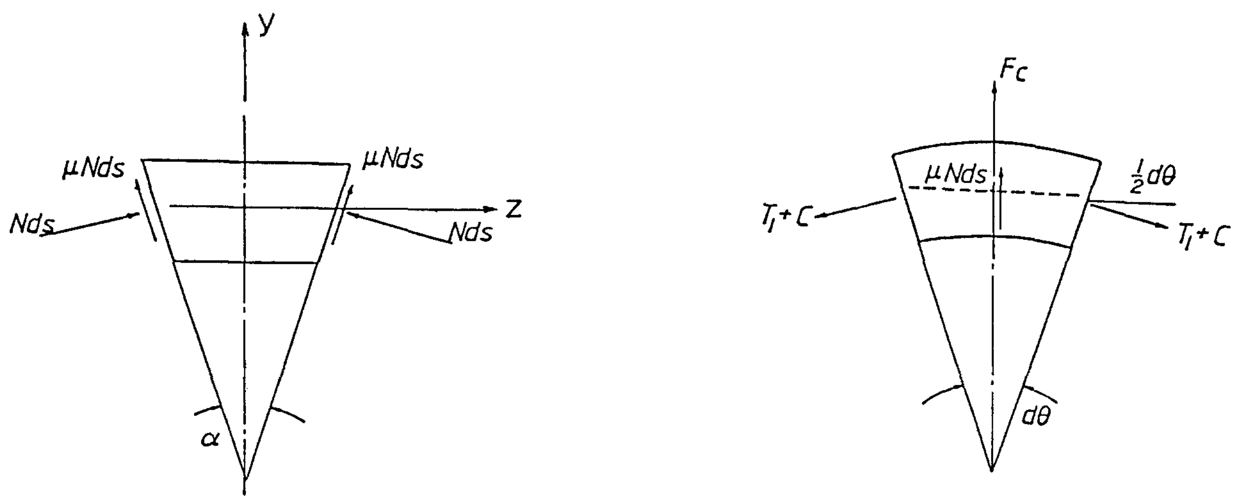

Fig. 4 Free body diagram of a driver pulley belt element

obtained from the axial force component in Z-direction.

$$Fds = Nds\left(\cos\frac{\alpha}{2} - \mu\sin\frac{\alpha}{2}\right) = \frac{T_1 d\theta\left(\cos\frac{\alpha}{2} - \mu\sin\frac{\alpha}{2}\right)}{2\left(\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\right)} \quad (3)$$

Integration of Eq.(3) over the entire arc of contact gives the driver pulley axial force.

$$F_R = \frac{T_1}{2}\theta\frac{1-\mu\tan\frac{\alpha}{2}}{\mu+\tan\frac{\alpha}{2}} \quad (4)$$

where θ is the arc of contact.

The driver pulley axial force, Eq.(4) has basically the same form with the Worley's equation except the coefficient of friction μ. In this paper, μ is constant while μ of the Worley's equation is expressed as $\mu=\sqrt{(\mu_r^2+\mu_t^2)}$ where μ_r is a radial component of friction and μ_t is a tangential component of friction. As shown in the equation(4), the driver pulley axial force is a function of the tight side belt tension T_1 and the arc of contact θ.

2.3 Driven Pulley Axial Force

The driven pulley axial force F_N is a summation of the axial force in the inactive area F_{Ni} and the axial force in the active area, F_{Na}.

$$F_N = F_{Ni} + F_{Na} \quad (5)$$

For the inactive area, there is no change in the belt tension. Thus, the same free body digaram of a belt element of Fig.4 can be used with the belt tension T_2 in stead of T_1. The axial force F_{Ni} can be derived in the same manner.

$$F_{Ni} = \frac{T_2\theta_i}{2}\left[\frac{1-\mu\tan\frac{\alpha}{2}}{\mu+\tan\frac{\alpha}{2}}\right] \quad (6)$$

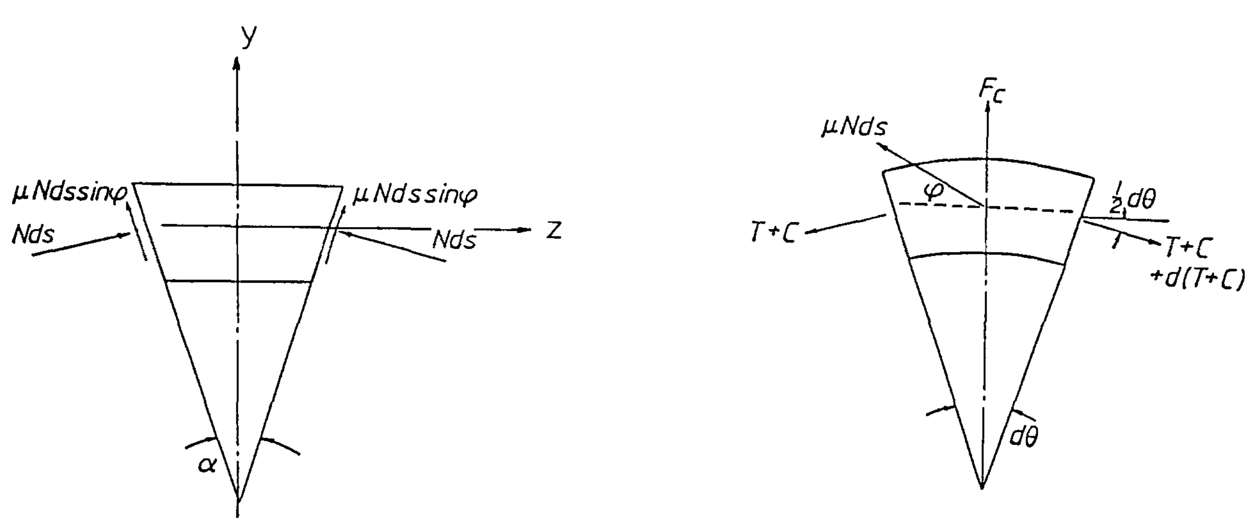

Fig. 5 Free body diagram of a driven pulley belt element

where θ_i is the magnitude of the inactive area.

For the active area, free body diagram of an element is shown on Fig.5. The belt moves in the radial and tangential direction and the friction occurs in the opposite direction of the belt movement. The angle ϕ between the friction and the tangent line is defined as 'sliding angle' following the definition of Oliver(1973). Radial and tangential equilibrium for the belt element requires

$$F_c + 2Nds\sin\frac{\alpha}{2} + 2\mu Nds\sin\phi\cos\frac{\alpha}{2} - (T+C)\sin\frac{\alpha}{2} - [(T+C)+d(T+C)]\sin\frac{d\theta}{2} = 0 \quad (7)$$

$$[(T+C)+d(T+C)]\cos\frac{d\theta}{2} - (T+C)\cos\frac{d\theta}{2} - 2\mu Nds\cos\phi = 0 \quad (8)$$

Simplifying the equation (7) and (8) gives

$$Td\theta = 2Nds(\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\sin\phi) \quad (9)$$

$$dT = 2\mu N\cos\phi ds \quad (10)$$

From Eq.(9) and (10),

$$\frac{dT}{T} = \frac{\mu\cos\phi\ d\theta}{\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\sin\phi} \quad (11)$$

Integrating Eq.(11) over the active arc gives

$$\frac{T_1}{T_2} = \exp\left[\frac{\mu\theta_a\cos\phi}{\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\sin\phi}\right] \quad (12)$$

where θ_a is the magnitude of the active area. The sliding angle ϕ is expressed as

$$\phi = arc\cos\left[\frac{1}{\mu\theta_a}(\ln\frac{T_1}{T_2})(\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\sin\phi)\right] \quad (13)$$

Axial force for the belt element, Fds is obtained from the axial force component in Z-direction.

$$Fds = Nds(\cos\frac{\alpha}{2} - \mu\sin\frac{\alpha}{2}\sin\phi) = \frac{dT}{2\mu\cos\phi}(\cos\frac{\alpha}{2} - \mu\sin\frac{\alpha}{2}\sin\phi) \quad (14)$$

Substituting Eq.(13) into Eq.(14) and integrating Eq.(14) over the active angle θ_a gives,

$$F_{Na} = \frac{(T_1 - T_2)\theta_a}{2\ln\frac{T_1}{T_2}} \frac{\cos\frac{\alpha}{2} - \mu\sin\frac{\alpha}{2}\sin\phi}{\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\sin\phi} \quad (15)$$

Now, we have two equations Eq.(13) and Eq. (15), and three unknowns F_{Na}, ϕ and θ_a. In order to obtain the axial force of the active area, we have to solve Eq.(13) and Eq.(15) by iterative numerical method. This is not the case that we expected to develop a simple analytical equation. If we know the sliding angle ϕ for a given tension ratio, we can solve Eq. (12) analytically for the active angle θ_a.

$$\theta_a = \frac{1}{\mu\cos\phi}(\ln\frac{T_1}{T_2})\ (\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\sin\phi) \quad (16)$$

Substituting Eq.(16) into Eq.(15), Eq.(15) becomes,

$$F_{Na} = \frac{T_1 - T_2}{2\mu\cos\phi}(\cos\frac{\alpha}{2} - \mu\sin\frac{\alpha}{2}\sin\phi) \quad (17)$$

Still, we have to determine the value of sliding angle ϕ for a given tension ratin to solve Eq.(17) for the axial force F_{Na}.

Gerbert(1972) presented a set of numerical solutions for the sliding angle ϕ, tension ratio, belt radial displacement with respect to the contact angle θ. Although Gerbert's solution was obtained based on different assumptions from those of this paper, close inspection of the Gerbert's numerical results provides an information to approximate the sliding angle ϕ. For the tension ratios over 2, which are mostly used operating conditions for the V-belt CVT drives, the sliding angle ϕ decreases from 30° to 21°, showing an asymptote to 20°, as the tension ratio increases(Gerbert, 1972). Thus, in this paper, we make another assumption that the sliding angle ϕ is zero. This makes Eq.(17) simple.

$$F_{Na} = \frac{T_1 - T_2}{2\mu}\cos\frac{\alpha}{2} \quad (18)$$

Now, the driven pulley axial force can be obtaind as a summation of the force in the inactive area, Eq.(6) and the force in the active area, Eq.(18)

$$F_N = \frac{T_2}{2}(\theta - \theta_a)\left(\frac{1 - \mu\tan\frac{\alpha}{2}}{\mu + \tan\frac{\alpha}{2}}\right) + \frac{T_1 - T_2}{2\mu}\cos\frac{\alpha}{2} \quad (19)$$

where $\theta - \theta_a = \theta_i$

Also, from the assumption of $\phi = 0°$ and Eq.(16), the magnitude of the active arc θ_a is,

$$\theta_a = \frac{1}{\mu}(\ln\frac{T_1}{T_2})\sin\frac{\alpha}{2} \quad (20)$$

The rationale of the above assumption is : comparing only the magnitude of the active angle of Eq.(16) with $\phi = 30°$ and Eq(20) of $\phi = 0°$, the error percentage between those of the two equations reaches almost 100%. However, the driven pulley axial force of the final equation(19) shows an error of 5% at maximum in spite of the difference in the active angle. This can be explained as follows : when the active angle is large, in other words, transmitted torque is large, the second term of Eq.(19) dominates the driven pulley axial force while the first term of Eq.(19) constitutes a large portion of the axial force when the active angle is small. Thus, the assumption of $\phi = 0°$ can be considered a good approximation.

Final results of the axial force equation, Eq.(4) and Eq.(19) are simple analytical equations. Since the magnitude of the belt tension and the arc of contact are directly related with the torque load and speed ratio, any axial force can be calculated easily by substituting belt tensions for any given speed ratio.

2.4 Nondimensional Axial Force Equation

In order to comapre the new formula for the V-belt CVT axial forces, equations(4) and (19), with the results of Worley, Oliver and Gerbert, Eq.(4) and (19) are nondimensionalized using a traction coefficient. The traction coefficient λ is defined as $\lambda=(T_1-T_2)/(T_1+T_2)$. Since torque load is proportional to the tension differnece T_1-T_2, λ increases as the torque load increases. Nondimensional form of Eq.(4) and (19) are,

$$\overline{F_R}=\frac{F_R}{T_1+T_2}=\frac{T_1\theta}{2(T_1+T_2)}\left[\frac{1-\mu\tan\frac{\alpha}{2}}{\mu+\tan\frac{\alpha}{2}}\right]=\frac{1+\lambda}{4}\theta\left[\frac{1-\mu\tan\frac{\alpha}{2}}{\mu+\tan\frac{\alpha}{2}}\right] \tag{21}$$

$$\overline{F_N}=\frac{F_N}{T_1+T_2}=\frac{T_2}{2(T_1+T_2)}(\theta-\theta_a)\left[\frac{1-\mu\tan\frac{\alpha}{2}}{\mu+\tan\frac{\alpha}{2}}\right]+\frac{T_1-T_2}{2\mu(T_1+T_2)}\cos\frac{\alpha}{2}=\frac{1-\lambda}{4}(\theta-\theta_a)\left[\frac{1-\mu\tan\frac{\alpha}{2}}{\mu+\tan\frac{\alpha}{2}}\right]+\frac{\lambda}{2\mu}\cos\frac{\alpha}{2} \tag{22}$$

3. RESULTS

Figure 6 shows nondimensional axial forces of the new formula, Eq.(21) and Eq.(22) with respect to the traction coeffieient λ. As input data, the coefficient of friction $\mu=0.4$ and the pulley wedge angle $\alpha=30^\circ$ are used for the speed rations $R=1$, 2, and 1/2(corresponding arc of contact for the driver pulley are $\theta=180^\circ$, 158°, 202°). As shown on Fig.6, the driver pulley axial force increases linearly with increasing traction coefficient λ(torque load). However, the driven pulley axial force does not change much compared with the driver pulley axial force in spite of the increasing torque load. This means that in the V-belt CVT car drive, the driver pulley axial force should be increased linearly for the increasing torque load to maintain a constant speed while the driven pulley axial force should be kept following the curves in Fig. 6. As the speed ratio increases from 1/2 to 2, the driver pulley axial force decreases and the driven pulley axial force increases. This can be explained by the magnitude of the contact arc. At the speed ratio 1/2, the driver pulley has the maximum contact arc for the given speed range that is from 1/2 to 2 in this paper. On the contrary, the driven pulley has the minimum contact arc at speed ratio 1/2. Since the axial force depends on the arc of contact, the driver pulley has the highest axial force and the driven pulley has the lowest axial force at speed ratio 1/2. As the speed ratio increases, the arc

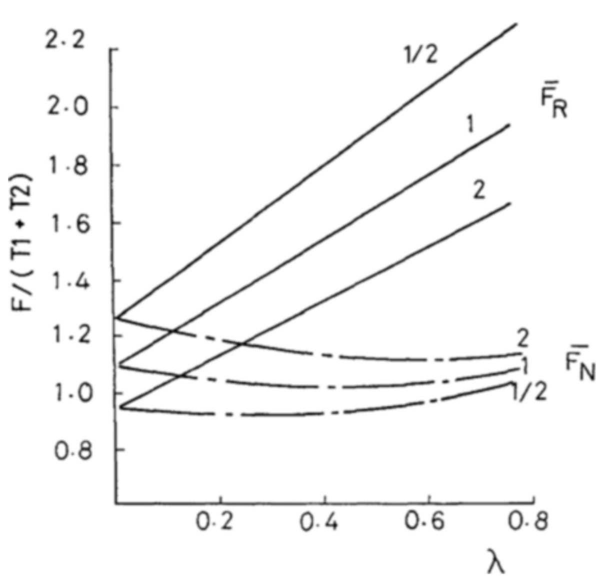

Fig. 6 Nondimensional axial forces for new formula

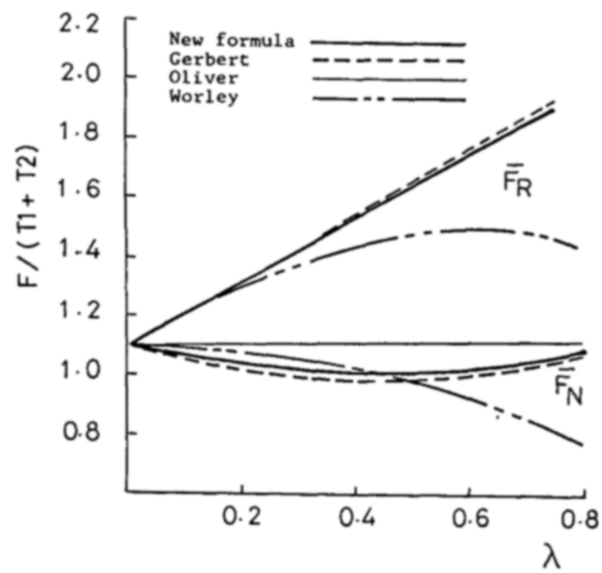

Fig. 7 Comparision of nondimensional axial forces for speed ratio 1

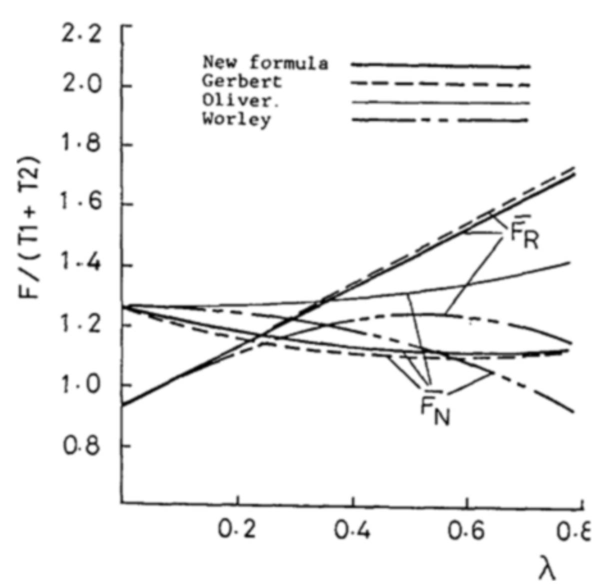

Fig. 8 Comparision of nondimensional axial forces for speed ratio 2

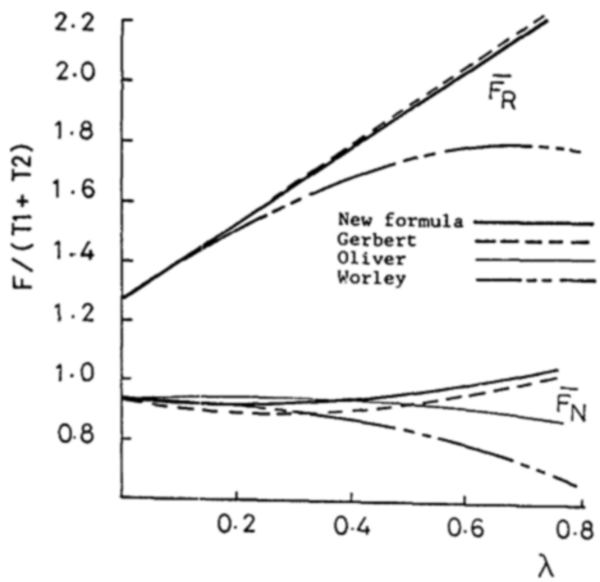

Fig. 9 Comparision of nondimensional axial forces for speed ratio 1/2

of contact of the driver pulley decreases and that of the driven pulley increase. Thus, the driver pulley axial force decreases and the driven pulley axial force increases.

In Figs. 7,8,9, the axial forces of the new formula are compared with those of Worly, Oliver and Gerbert for speed ratio $R=1,2$ and 1/2. As Fig.7 indicates, for the speed ratio $R=1$, the driver pulley axial force of the new formula is close to the exact solution of Gerbert. Only the gradient of the curve by the new equation is slightly lower than that of Gerbert. Oliver's solution for the driver pulley axial force shows virtually identical results with the new formula's. However, Worley's equations underestimates the axial force and this underestimate increases as λ increases. For the driven pulley axial force, the general shape and trend of the new formula agree well with Gerbert's. Both curves have concave shape, while Oliver's curve remains almost constant. Worley's curve decreases as the traction coefficient increases. At the larger value of λ, the difference from the exact solution becomes greater in the Worley's curve.

As for the speed ratio $R=2$ and 1/2, similar trends can be observed in Fig.8 and Fig.9. The axial force of the new formula shows close agreement with the exact solution by Gerbert in both the driver and the driven pulley. For the axial force curve of Oliver, the driver pulley axial force agree well with Gerbert's. But in the driven pulley axial force, it shows a difference and the difference from the exact solution becomes larger at the larger value of λ. For the axial force curve of Worley, both the driver and the driven pulley axial force do not agree with the exact solution by Gerbert. The difference between the Worley's axial force and the Gerberts' increase with the increasing traction coefficient λ. Since most V-belt drives are operated at large traction coefficient to improve the efficiency, it is very important to have an exact relationship between the axial force and the torque load for given speed ratio.

Comparing the solutions obtained from the new formula, Eq.(21) ad Eq.(22) with the results by the previous works, it is found that the results of the new formula are in close agreement with the exact solution by Gerbert for the driver and driven pulley axial force. The difference between the solution of the new formula and the exact solution is 5% at maximum, which is tolerable in the engineering sense.

Therefore, considering the simplicity and accuracy of the new formula, it is suggested to use the new formula for the V-belt CVT design.

4. CONCLUSION

Simple analytical equations were dervied for the axial forces of V-belt CVT. At any given torque load and speed ratio, the results of the new equation were in good agreement with the exact solutions. The new equation is suggested as a design formula for the V-belt CVT design due to its simplicity and sufficient accuracy.

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